〈論　文〉

A Note on Price and Quantity Competitions in Buyer-Seller Networks

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Abstract: Manufacturers and suppliers establish business connections. Suppliers compete for customers that are manufacturers. We model as a two-stage game the price competition among suppliers chosen by manufacturers. In the first stage, the suppliers produce. In the second stage, the suppliers determine prices for buyers (manufacturers) without information of exact buyers' valuations, and buyers choose suppliers simultaneously. We describe the choices as links, and a network is composed of the links. This market is called a networked market, and we show that buyers' linking depends on the link cost and production amounts. Manufacturers do not choose the lower production supplier because manufacturers are afraid of selling out.

Keywords: Price competition; Network formation;
Journal of Economic Literature Classication Numbers: C72; D85

1　Introduction

In many industries, manufacturers and suppliers have particular business relationships. Manufacturers produce goods by adjusting the components produced by suppliers. For example, in the current CPU market, Intel and AMD, which are suppliers of the CPU, were said to be experiencing a price competition. Their customers are manufacturers of computer hardware, such as Dell, HP, and Apple. Once a manufacturer adopts a component for its product, it typically uses the component for a long time because it is difficult to imeediately switch out the component for one from another manufacture. Therefore, for a manufacturer, deciding on the components to adopts is important. For a supplier, the number of customers that it secures is important. We consider a relationship between manufacturer and supplier as a link, such links consist of a network, and a market described as a network is called a networked market.

The focus of the literature on networked markets is how networks affect the prices and efficiency of a market, and how a position in a network affects the trade and welfare of an agent. In a series of papers, Kranton and Minehart (2000a, b, and 2001) analyzed the formation of buyer-seller networks. Kranton and Minehart (2001) studied goods auctioned off in networks. In their model, the price at which markets clear is competitive, and thus, the allocation as a result of trading is efficient. Corominas-Bosh (2004) studied bargaining between buyers and sellers connected by a network and modeled the bargaining by an alternating move game between buyers and sellers.

The present paper models price competitions in buyer–seller networks. Buyers choose components they adopt, and their choices are described by links, each of which indicates that a buyer adopts a seller's good. Sellers are engaged in price competition with quantity precommitments. Sellers do not know buyers' exact valuations for goods. Then, we show that equilibrium prices decrease even though buyers adopt one seller. Intuitively, if the cost of forming a link is small, then buyers may have the incentive to form multiple links. This incentive is derived from the possibility that if many buyers choose the same seller, then buyers cannot obtain the good by selling out. Hence, if sellers produce a number of the good, then buyers form only one link because they can certainly buy the good.

This paper is closely related to the industrial organization literature on capacity compe- tition. For example, Kreps and Sheinkman (1983) showed the result that *quantity precom- mitment and Bertrand competition yield Cournot outcomes* is well known.[1)]

In the next section, we introduce the definitions. Section 3 studies the price competition game in buyer–seller networks with quantity precommitments. Section 4 compares the prices determined by price competition and no competition. Section 5 concludes.

2 Model

2. 1 Players and goods

There are two buyers and two sellers. Let $\mathcal{B} = \{b_1, b_2\}$ be the set of *buyers* who each demand only one indivisible unit of good. Let $\mathcal{S} = \{s_1, s_2\}$ be the set of *sellers* who each sell good j. Each buyer b_i has valuations v^i_j for good j. Let $\mathcal{V}_j = [0, 1]$, a closed interval, denote the set of valuations for good j. Let $\mathcal{V} = \mathcal{V}_1 \times \mathcal{V}_2$. Let $v^i = (v^i_1, v^i_2)$ be a profile of buyer b_i's valuations. Let $\mathbf{v} = (v^1, v^2)$ denote a profile of buyers' valuation profiles.

2. 2　Networks

Buyers and sellers can angage in exchange only if they are linked. Let $g_{ij} \in \{0, 1\}$ denote a relationship between buyer b_i and seller s_j. If b_i and s_j are linked, then $g_{ij} = 1$, and 0 otherwise. Let G be a 2×2 matrix, where the (i, j)th element is g_{ij}. We denote a *network* by G. Let $\mathcal{G}$ be the set of all networks. For a given network G and a set of buyers $\mathcal{B}' \subseteq \mathcal{B}$,

$$\text{let } L(\mathcal{B}') = \{j \in \mathcal{S} : i \in \mathcal{B}' \text{ and } g_{ij} = 1\}$$

denote the buyers' *linked* set of sellers. Similarly, for a set of sellers $\mathcal{S}' \subseteq \mathcal{S}$, let $L(\mathcal{S}') = \{i \in \mathcal{B} : j \in \mathcal{S}' \text{ and } g_{ij} = 1\}$ denote the sellers' *linked set* of buyers.

2. 3　Linking and pricing

Buyer b_i's valuation for good j is distributed over the interval $\mathcal{V}_j = [0, 1]$ according to the uniform distribution function $F : \mathcal{V}_j \to [0, 1]$ with the associated density function f. All distributions are common knowledge. The valuations are private information for the buyers. Sellers do not know the exact valuations.

Let $\lambda_i : \mathcal{V} \to \{0, 1\}^2$ be the *linking* of buyer b_i, denoted by

$$\lambda_i(v^i) = (\lambda_{i1}(v^i), \lambda_{i2}(v^i)) \in \{0, 1\}^2,$$

where $\lambda_{ij}(v^i) = 1$ indicates that b_i can adopt seller s_j's good as input when b_i's valuation is v^i. The network formed in this stage is denoted by a 2×2 matrix $G = [\lambda_{ij}(v^i)]$. Let $\boldsymbol{\lambda} = (\lambda_1, \lambda_2)$ denote a profile of buyers' linkings, and let Λ be the set of linkings, forming a link cost d for the buyer. We assume that $d \leq 1/4$. Therefore, buyer b_i pays the cost $\sum_{j=1}^{2} \lambda_{ij}(v^i)d$.

Let $p_j \in \mathbb{R}^2_+$ be seller s_j's *pricing*, which is denoted by $p_j = (p_{1j}, p_{2j})$, where p_{ij} is a price put by seller s_j for b_i. Let $\mathbf{p} = (p_1, p_2)$ denote a profile of all sellers' pricings.

2. 4　Payoffs

We now define buyers' and sellers' payoff functions. First, we define the buyer's payoff function. If buyer b_i purchases a good j, then the net benefit from good j is $v^i_j - p_{ij}$. Furthermore, buyer b_i pays a linking cost xa, where x is the number of links formed by b_i Then, b_i's payoff is $v^i_j - p_{ij} - xa$.

The *buyer's expected payoff function* is $V_i : \Lambda \times \mathbb{R}^2_+ \times V \times \mathbb{Z}^2_+ \to \mathbb{R}$, defined by

$$V_i(\boldsymbol{\lambda}, \mathbf{p}; v^i, \mathbf{q}) = \int_V \beta_{ij}(\boldsymbol{\lambda}, \mathbf{p}, v^{-i} \mid v^i, \mathbf{q})(v^i_j - p_{ij}) f^{(2)}(v^{-i}) dv^{-i} - \sum_{j=1}^{2} \lambda_{ij}(v^i)d,$$

where $\beta_{ij}(\boldsymbol{\lambda}, \mathbf{p}, v^{-i} \mid v^i, \mathbf{q})$ is the probability that buyer b_i buys from seller s_j. The probability β_{ij} is defined in the next section.

The *seller's expected profit function* is $\Pi_j : \Lambda \times \mathbb{R}^2_+ \times \mathbb{Z}^2_+ \to \mathbb{R}$, defined by

$$\Pi_j(\boldsymbol{\lambda}, \mathbf{p}; \mathbf{q}) = \int_{V^{(2)}} \theta_{ij}(\boldsymbol{\lambda}, \mathbf{p}, \mathbf{v} \mid \mathbf{q}) p_{ij} f^{(4)}(\mathbf{v}) d\mathbf{v} - cq_j,$$

where $\theta_{ij}(\boldsymbol{\lambda}, \mathbf{p}, \mathbf{v} \mid \mathbf{q})$ is a probability that seller s_j sells a good to buyer b_i, and c is the cost of producing one unit. The probability θ_{ij} is defined in the next section.

3 The game

We now consider the game in which each buyer's strategy is to form links and each seller's is production and pricing. We define a two-stage game. In Stage 1, each seller determines its production amounts. In Stage 2, buyers and sellers move simultaneously with knowledge of how many units sellers produce. Buyers choose a seller (sellers) from which they want to buy. Sellers determine prices for buyers individually.

The game is solved in a backward manner. At first, we solve the simultaneous move game in Stage 2. We introduce two strategies.

$$\lambda^{\mathrm{I}}_{ij}(v^i) = \begin{cases} 1 & \text{if } v^i_j - p_{ij} > v^i_{j'} - p_{ij'} \text{ and } v^i_j \geq p_{ij} + d, \\ 0 & \text{otherwise,} \end{cases}$$

where if $v^i_j - p_{ij} = v^i_{j'} - p_{ij'}$, then $\lambda^{\mathrm{I}}_{ii}(v^i) = 1$ and $\lambda^{\mathrm{I}}_{ik}(v^i) = 0, k \neq i$. If buyer b_i takes strategy λ^I_i, b_i forms at most one link that connects to a seller that b_i prefers.

Let λ^{II}_i be

$$\lambda^{\mathrm{II}}_i(v^i) = \begin{cases} (1,1) & \text{if } v^i_j \geq p_{ij} + 2d, j = 1, 2 \\ (1,0) & \text{if } v^i_1 \geq p_{i1} + d, v^i_2 < p_{i2} + 2d \text{ and } v^i_1 - p_{i1} > v^i_2 - p_{i2} \\ (0,1) & \text{if } v^i_1 < p_{i1} + 2d, v^i_2 \geq p_{i2} + d \text{ and } v^i_2 - p_{i2} > v^i_1 - p_{i1} \\ (0,0) & \text{otherwise,} \end{cases}$$

where if $v^i_j - p_{ij} = v^i_{j'} - p_{ij'}$, $v^i_j \geq p_{ij} + d$, and $v^i_{j'} < p_{ij'} + 2d$, then $\lambda^{\mathrm{II}}_{ii} = 1$ and $\lambda^{\mathrm{II}}_{ik} = 0, k \neq i$.
If buyer b_i takes strategy λ^{II}_i, b_i forms at most two links.

We say that buyer b_i's link that connects to j satisfies the *single formation constraint* if $v^i_j \geq p_{ij} + d$, that is, buyer b_i does not lose if it buys a good. Furthermore, we say that b_i's links satisfy the *full formation constraint* if $v^i_j \geq p_{ij} + 2d$ for $j = 1, 2$, that is, buyer b_i does not lose if it buys a good.

3. 1　The case that each seller has one unit

Let each seller have one unit of the good. We show that buyers' linkings depend on the link cost. Section 3.1.1 investigates the case that each buyer forms at most one link. Section 3.1.2 investigates the case that each buyer forms at most two links.

3. 1. 1　Each buyer forms at most one link

We focus on a symmetric equilibrium in which buyers form at most one link and sellers use the

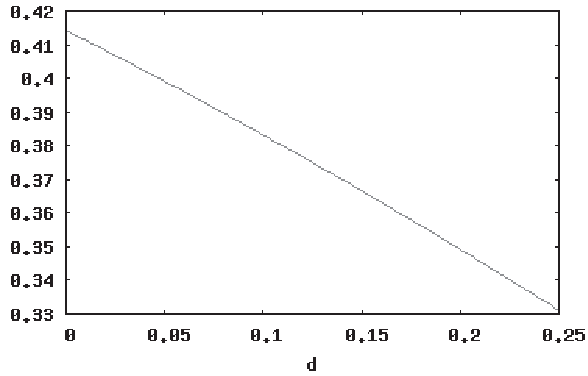

Figure 1: The price that maximizes the expected revenue respective to d.

same price. Seller s_1 sets prices for buyers without knowledge of their exact valuations.
The probability that buyer b_1 forms a link connecting to s_1 is as follows. If $v_1^1 - p_{11} > v_2^1 - p_{12}$, then b_1 forms the link. Thus,

$$\begin{array}{ll} (1-p_{11}-d) - \dfrac{(1-p_{12}-d)^2}{2} & \text{if } p_{11} \le p_{12}, \text{and} \\ \dfrac{(1-p_{11}-d)(-p_{11}+2p_{12}+1+d)}{2} & \text{if } p_{11} > p_{12}. \end{array}$$

Then, s_1's expected revenue is

$$\begin{array}{ll} p_{11}(1-p_{11}-d) - p_{11}\dfrac{(1-p_{12}-d)^2}{2} & \text{if } p_{11} \le p_{12}, \quad \text{and} \\ p_{11}\dfrac{(1-p_{11}-d)(-p_{11}+2p_{12}+1+d)}{2} & \text{if } p_{11} > p_{12}. \end{array}$$

Therefore, sellers' best responses are

$$\begin{aligned} p_{11}(p_{12}) &= -\frac{p_{12}^2 + p_{12}(2d-2) + d^2 - 1}{4}, \quad \text{and} \\ p_{12}(p_{11}) &= -\frac{p_{11}^2 + p_{11}(2d-2) + d^2 - 1}{4}. \end{aligned}$$

Thus, in equilibrium, they set the price at $p_{11} = p_{12} = \sqrt{2(d+1)} - (d+1)$. Let $p^* = \sqrt{2(d+1)} - (d+1)$ denote the equilibrium price. Figure 1 shows how sellers determine the prices according to the link cost.

Next, we consider buyers' linkings and fix b_1's valuation v^1. Suppose that $v_1^1 < p^* + d$ and $v_2^1 < p^* + d$. Buyer b_1 forms no link. Then, we have

$$V_1(\boldsymbol{\lambda}^{\mathrm{I}}, \mathbf{p}^*; v^1, \mathbf{1}) = 0\,,$$

where $\mathbf{1} = (1, 1)$ denotes that each seller has one unit. Clearly, if b_1 forms a link, then b_1's payoff is negative. Therefore,

$$V_1(\boldsymbol{\lambda}^{\mathrm{I}}, \mathbf{p}^*; v^1, \mathbf{1}) \ge V_1(\lambda_1, \lambda_2^{\mathrm{I}}, \mathbf{p}^*; v^1, \mathbf{1})$$

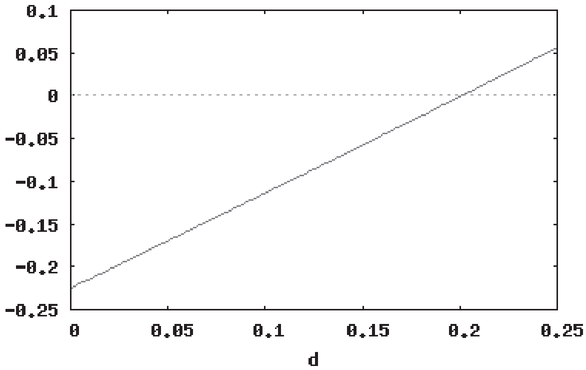

Figure 2: The deviation loss by forming at most two links

for all linkings λ_1.

Next, we suppose that $v_1^1 - p^* > v_2^1 - p^*$ and $v_1^1 \geq p^* + d$. Then, we have

$$\begin{aligned}&V_1(\boldsymbol{\lambda}^{\mathrm{I}}, \mathbf{p}^*; v^1, \mathbf{1})\\ &= \left(1 - \frac{1-(p^*+d)^2}{2}\right)(v_1^1 - p^*) + \frac{1-(p^*+d)^2}{4}(v_1^1 - p^*) - d.\end{aligned}$$

Suppose that b_1 takes strategy λ_1^{II}. If $v_2^1 < p^* + 2d$, then b_1 forms one link that connects to seller s_1. Therefore, b_i's expected payoff is the same as that previously described. If b_1 forms two links, that is, $v_j^1 \geq p^* + 2d$ for all $j = 1, 2$, then b_1's expected payoff is

$$\begin{aligned}&V_1(\lambda_1^{\mathrm{II}}, \lambda_2^{\mathrm{I}}, \mathbf{p}^*; v^1, \mathbf{1})\\ &= \left(1 - \frac{1-(p^*+d)^2}{2}\right)(v_1^1 - p^*)\\ &\quad + \frac{1-(p^*+d)^2}{4}(v_1^1 - p^*) + \frac{1-(p^*+d)^2}{4}(v_2^1 - p^*) - 2d.\end{aligned}$$

Thus,

$$\begin{aligned}&V_1(\boldsymbol{\lambda}^{\mathrm{I}}, \mathbf{p}^*; v^1, \mathbf{1}) - V_1(\lambda_1^{\mathrm{II}}, \lambda_2^{\mathrm{I}}, \mathbf{p}^*; v^1, \mathbf{1})\\ &= d - \frac{1-(p^*+d)^2}{4}(v_2^1 - p^*).\end{aligned}$$

Since $v_2^1 \leq 1$, it holds that

$$\begin{aligned}d - \frac{1-(p^*+d)^2}{4}(v_2^1 - p^*) &\geq d - \frac{1-(p^*+d)^2}{4}(1 - p^*)\\ &= -\frac{-2d^2 + \sqrt{2(d+1)}\,(4d+7) - 15d - 9}{4}.\end{aligned}$$

Therefore, if the link cost, d, is sufficiently large, then the buyer has no incentive to form two links. Figure 2 shows that if d is sufficiently large (e.g., $d > 0.21$), then $V_1(\boldsymbol{\lambda}^{\mathrm{I}}, p^*; v^1, \mathbf{1}) - V_1(\lambda_1^{\mathrm{II}}, \lambda_2^{\mathrm{I}}, p^*; v^1, \mathbf{1})$ is positive. Buyer b_2's also holds. Then, an equilibrium exists in which each buyer forms at most one link and each seller sets the price at $\sqrt{2(d+1)} - (d+1)$ if the link cost is

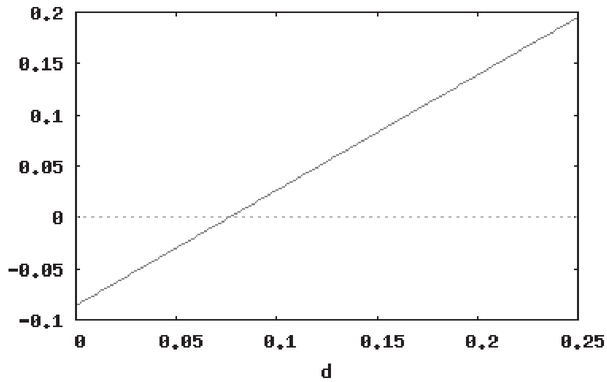

Figure 3: The deviation loss by forming at most two links

sufficiently large.

Conversely, since $v_2^1 > p^* + 2d$, we have

$$
\begin{aligned}
& V_1(\boldsymbol{\lambda}^{\mathrm{I}}, \mathbf{p}^*; v^1, \mathbf{1}) - V_1(\lambda_1^{\mathrm{II}}, \lambda_2^{\mathrm{I}}, \mathbf{p}^*; v^1, \mathbf{1}) \\
&= d - \frac{1-(p^*+d)^2}{4}(v_2^1 - p^*) \\
&< d - \frac{1-(p^*+d)^2}{4}p^* \\
&= d - \frac{(\sqrt{2(d+1)} - (d+1))^2}{2}
\end{aligned}
$$

Thus, if d is sufficiently small, then buyer b_1 has incentive to form two links. Figure 3 shows that if the link cost d is sufficiently small (e.g., $d < 0.075$), then $d - (\sqrt{2(d+1)} - (d+1))^2/2$ is negative, and b^1 deviates from $(\boldsymbol{\lambda}^{\mathrm{I}}, p^*)$.

3. 1. 2 Each buyer forms at most two links

We show that if each seller has one unit of the good, then each buyer never forms two links. Because each seller sets prices individually, its pricing is the same as that in the case studied. That is, sellers name their goods $\sqrt{2(d+1)} - (d+1)$.

We consider buyers' linkings. Suppose that $v_1^1 > p^* + 2d$, $v_2^1 > p^* + 2d$, and $v_1^1 - p^* > v_2^1 - p^*$. Then, b_1's expected payoff is

$$
\begin{aligned}
& V_1(\boldsymbol{\lambda}^{\mathrm{II}}, \mathbf{p}^*; v^1, \mathbf{1}) \\
&= \left(1 - \frac{1-(p^*+d)^2}{2}\right)(v_1^1 - p^*) \\
&\quad + \frac{1-(p^*+d)^2}{4}(v_1^1 - p^*) + \frac{1-(p^*+d)^2}{4}(v_2^1 - p^*) - 2d.
\end{aligned}
$$

If b_1 takes linking λ_1^{I}, then its expected payoff is

$$V_1(\lambda_1^{\mathrm{I}}, \lambda_2^{\mathrm{II}}, \mathbf{p}^*; v^1, \mathbf{1})$$
$$= \left(1 - \frac{1 - (p^* + d)^2}{2}\right)(v_1^1 - p^*) + \frac{1 - (p^* + d)^2}{4}(v_1^1 - p^*) - d.$$

Again, if the link cost is sufficiently small, then

$$V_1(\boldsymbol{\lambda}^{\mathrm{II}}, \mathbf{p}^*; v^1, \mathbf{1}) > V_1(\lambda_1^{\mathrm{I}}, \lambda_2^{\mathrm{II}}, \mathbf{p}^*; v^1, \mathbf{1}).$$

Thus, it is an equilibrium in which each buyer forms at most two links and each seller set the price at $\sqrt{2(d+1)} - (d+1)$. However, if the link cost is sufficiently small, then it is not an equilibrium.

3. 2 The case in which each seller has two units

Because sellers offer individual prices for buyers, the price that maximizes sellers' expected revenue is the same as the case previously discussed. That is, each seller sets the price at $\sqrt{2(d+1)} - (d+1)$.

We consider buyers' linkings. Now, sellers have two units. Therefore, if a buyer connects a seller from which the buyer wants to buy, then it can necessarily buy a good from the seller. Therefore, each buyer has no incentive to form two links. Thus, in equilibrium, if each seller has two units, buyers bi take strategies λ_i^{I}.

3. 3 The case that seller s_1 has one unit and s_2 has two units

We investigate the case in which seller s_1 has one unit and s_2 has two units of a good. In this case, if a buyer wants to buy s_2's good, then the buyer can necessarily buy the good. Therefore, the problem arises when the valuation for s_1' s good is higher than that of s_2' s, that is, $v_1^i > v_2^i$. As shown in Sections 3.1 and 3.2, if the link cost is sufficiently large, then buyers never form two links; and if the link cost is sufficiently small, then buyers form at most two links.

3. 4 Stage 2 results

We summarize the results obtained in the previous sections as the following proposition.

Proposition 1. In a symmetric equilibrium, each seller sets the price at $\sqrt{2(d+1)} - (d+1)$ and

(i) if the link cost, d, is sufficiently large, then in the equilibrium, each buyer forms at most one preferred link and satisfies the single formation constraint, or

(ii) if the link cost, d, is sufficiently small, then in the equilibrium, each buyer forms a link that is preferred and satisfies the single formation constraint and forms two links if the links satisfy the full formation constraint.

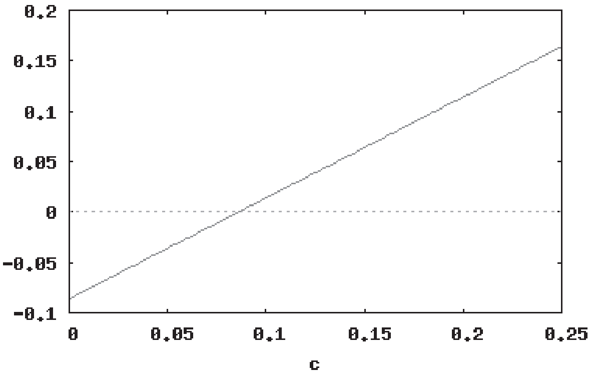

Figure 4: The deviation loss by producing two units

3. 5 How many units sellers produce?

3. 5. 1 The case that *d* is suffciently large

As shown in the previous sections, if the link cost is sufficiently large, then buyers form at most one link. Then, how many units do sellers maximize their expected profits? In the following, we show that the answer depends on the production cost. If the production cost is sufficiently large, then each seller produces only one unit.

If $v_1^1 > p^* + d$ and $v_1^1 > v_2^1$ or $v_1^2 > p^* + d$ and $v_1^2 > v_2^2$, then seller s_1 can sell a good.

Thus, seller s_1's expected profit is

$$\begin{aligned}\Pi_1(\boldsymbol{\lambda}^{\mathrm{I}}, \mathbf{p}^*; \mathbf{1}) &= \frac{(1-(p^*+d))^2}{2} p^* - c\\ &= d^2 - 2(d+1)\sqrt{2(d+1)} + 4d + 3 - c.\end{aligned}$$

If s_1 produces two units, then s_1's expected profit is

$$\begin{aligned}&\Pi_1(\boldsymbol{\lambda}^{\mathrm{I}}, \mathbf{p}^*; 2, 1)\\ &= p^* \frac{(1-(p^*+d))^4}{2} + (1-(p^*+d))^2 \left(1 - \frac{(1-(p^*+d))^2}{2}\right) p^* - 2c.\end{aligned}$$

Thus, we have

$$\begin{aligned}&\Pi_1(\boldsymbol{\lambda}^{\mathrm{I}}, \mathbf{p}^*; \mathbf{1}) - \Pi_1(\boldsymbol{\lambda}^{\mathrm{I}}, \mathbf{p}^*; 2, 1)\\ &= c - p^* \frac{(1-(p^*+d))^4}{2}\\ &\quad + \frac{(1-(p^*+d))^2}{2}(1 + (1-(p^*+d))^2) - (1-(p^*+d))^2\\ &= c + 2d^3 - \sqrt{2(d+1)}(10d^2 + 68d + 80) + 48d^2 + 153d + 113\end{aligned}$$

Now, the link cost d is sufficiently large. For example, let $d = 0.21$. If the production cost c is sufficiently large (e.g., $c = 0.2$), then $c + 2d^3 - \sqrt{2(d+1)}\,(10d^2 + 68d + 80) + 48d^2 + 153d + 113$ is necessarily positive, as shown in Figure 4. Thus, seller s_1 does not produce two units. The same is

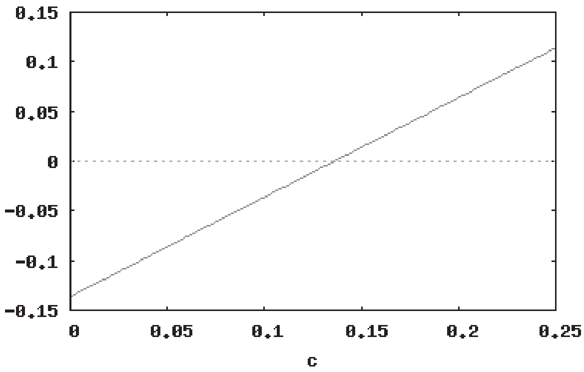

Figure 5: The deviation loss by producing two units

the case for seller s_2. Therefore, if c is sufficiently large, then each seller produces only one unit in Stage 1.

If each seller has two units, then s_1's expected profit is

$$\Pi_1(\boldsymbol{\lambda}^{\mathrm{I}}, \mathbf{p}^*; \mathbf{2}) = p^* \frac{(1-(p^*+d))^4}{2} + (1-(p^*+d))^2 \left(1 - \frac{(1-(p^*+d))^2}{2}\right) p^* - 2c.$$

If s_1 produces one unit, then s_1's expected profit is

$$\Pi_1(\boldsymbol{\lambda}^{\mathrm{I}}, \mathbf{p}^*; 1, 2) = d^2 - 2(d+1)\sqrt{2(d+1)} + 4d + 3 - c.$$

Thus, as shown in this equation, if c is sufficiently small, then $\Pi_1(\boldsymbol{\lambda}^{\mathrm{I}}, \mathbf{p}^*; \mathbf{2}) > \Pi_1(\boldsymbol{\lambda}^{\mathrm{I}}, \mathbf{p}^*; 1, 2)$. The same is true of seller s_2. Therefore, if the production cost is sufficiently small, then each seller produces two units in Stage 1.

3. 5. 2 The case that *d* is suffcientlysmall

If the link cost is sufficiently small, then buyers form at most two links. However, if each seller has two units, then buyers never form two links as shown in Section 3.2. Therefore, we investigate that whether $(\boldsymbol{\lambda}^{\mathrm{II}}, \mathbf{p}^*; \mathbf{1})$ is an equilibrium. If buyer's valuations are sufficiently high, then sellers form two links and obtain a further profit-taking opportunity. Thus,

$$\Pi_1(\boldsymbol{\lambda}^{\mathrm{II}}, \mathbf{p}^*; \mathbf{1}) = p^* \frac{(1-(p^*+d))^2}{2} + p^* \frac{(1-(p^*+2d))^4}{4} - c.$$

If seller s_1 produces two units, then s_1's expected profit is

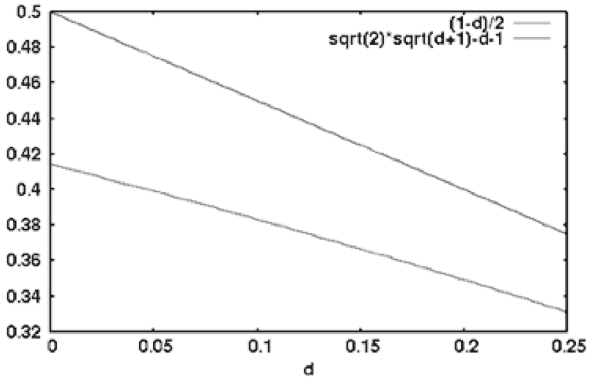

Figure 6: The prices with two sellers (the lower red line) and with one seller (the higher blue line).

$$\begin{aligned}&\Pi_1(\boldsymbol{\lambda}^{\mathrm{II}}, \mathbf{p}^*; 2, 1)\\ &= p^*\frac{(1-(p^*+d))^4}{2} + (1-(p^*+d))^2\left(1-\frac{(1-(p^*+d))^2}{2}\right)p^*\\ &\quad + p^*(1-(p^*+d))^2\frac{(1-(p^*+2d))^2}{2} - 2c\end{aligned}$$

Thus,

$$\begin{aligned}&\Pi_1(\boldsymbol{\lambda}^{\mathrm{II}}, \mathbf{p}^*; \mathbf{1}) - \Pi_1(\boldsymbol{\lambda}^{\mathrm{II}}, \mathbf{p}^*; 2, 1)\\ &= c + 2d^3 - \sqrt{2(d+1)}(10d^2+68d+80) + 48d^2 + 153d + 113\\ &\quad - \frac{d^5 + \sqrt{2(d+1)}(3d^4 - 16d^3 + 72d^2 + 256d + 116)}{4}\\ &\quad - \frac{(3d^4 - 12d^3 + 264d^2 + 444d + 164}{4}\\ &= c - \frac{d^5 + \sqrt{2(d+1)}(3d^4 - 16d^3 + 112d^2 + 528d + 436)}{4}\\ &\quad - \frac{3d^4 - 4d^3 + 456d^2 + 1056d + 616}{4}.\end{aligned}$$

Now, the link cost d is sufficiently small. For example, let $d = 0.05$. Then, if the production cost c is sufficiently large (e.g., $c = 0.2$), then $\Pi_1(\boldsymbol{\lambda}^{\mathrm{II}}, \mathbf{p}^*; \mathbf{1}) > \Pi_1(\boldsymbol{\lambda}^{\mathrm{II}}, \mathbf{p}^*; 2, 1)$, as shown in Figure 5. The same is true of seller s_2. Thus, if the production cost is sufficiently large, then each seller produces one unit.

3. 6 Equilibrium

We summarize the results obtained in the previous sections as the following proposition.

Proposition 2. In an symmetric equilibrium,

(i) If the link cost and production costs are sufficiently large, then each buyer forms at most one link that is preferred and that satisfies the single formation constraint, and each seller

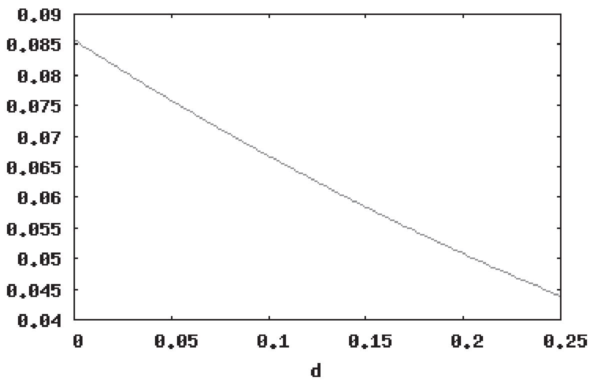

Figure 7: The difference between the price with two sellers and one with one seller.

produces one unit of a good,

(ii) If the link cost is sufficiently large and the production cost is sufficiently small, then each buyer forms at most one link that is preferred and that satisfies the single formation constraint, and each seller produces two units of good and

(iii) If the link cost is sufficiently small and the production cost is sufficiently large, then each buyer forms a link that is preferred, that satisfies the single formation constraint, and that forms two links if they satisfy the full formation constraint, and each seller produces one unit of the good.

4 Comparison with the case without competition

If the number of sellers is one, no competition occurs. A seller sets the price to maximize its expected profit. If $v_1^1 > p_{11} + d$, then buyer b_1 buys a good. Thus, its probability is $1 - F(p_{11} + d)$, and s_1's expected revenue is $p_{11}(1 - F(p_{11} + d))$. Then, the price that maximizes the expected profit is $(1 - d)/2$. As shown in the previous section, if the number of sellers is two, then the price is $\sqrt{2(d+1)} - (d+1)$. We have,

$$\frac{1-d}{2} - (\sqrt{2(d+1)} - (d+1)) > 0.$$

In Figure 6, the higher line depicts the price when only one seller exists, and the lower line depicts the price when two sellers exist. If the link cost d increases, then the difference between the prices decreases, which is shown in Figure 7. Thus, we have the following proposition.

Proposition 3. Price-quantity competition decreases the equilibrium price. Furthermore, if the link cost d increases, then the difference between prices with and without competition decreases.

5 Concluding remarks

This paper studies price competition in buyer–seller networks with quantity precommitment. We show that three types of symmetric equilibria exist. First, if both the link and the production cost are large, then each buyer forms at most one link and each seller produces one unit. Second, if the link cost is large, but the production cost is small, then each buyer forms at most one link and each seller produces two units. If each seller has two units, then each buyer certainly obtains the good it prefers. Thus, buyers have no incentive to form two links if sellers have two units. Furthermore, the equilibrium price with two sellers is lower than that with one seller. Last, if the link cost is small, but the production cost is large, then each buyer forms at most two links and each seller produces one unit. In the equilibrium, since each seller produces one unit if buyers' choices of the seller are the same, at least one buyer cannot obtain the good. Then, if the link cost is small, then each buyer forms two links to obtain certainly a good.

This paper developed a theory of networked markets in which buyers can obtain a good from a seller only if the two are linked. Our model is different from the two models introduced by Kranton and Minehart (2001), and Corominas-Bosch (2004). However, our study does not treat the general case of n agents. Furthermore, it is necessary that repeated situations study the behavior of price by price competition, which remains a matter for further discussion.

Annotation

1) Levitan and Shubik (1980), and Davidson and Deneckere (1990) also study such a market.

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